The Signal-Parametrical Invariance as a New Direction of the Disturbance Decoupling Problem

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***Abstract* — A system with weekly depending on effects and external factors is one of the major problems for engineering. There are a lot of approaches to synthesis of such systems, but essential progress in the decision is not present yet. In the paper, the concept of signal-parametrical invariance (disturbance decoupling) of nonlinear system design and application of this concept as optimization criterion is investigated. Conditions of invariance of the response system parameters to the effect amplitude are determined. Connection of classical and parametrical invariance is considered. Expression for defect of invariance is received and directions of it reduction are considered. Optimization by criterion of signal-parametrical invariance is discussed.**

***Index Terms* — Signal-parametrical invariance, disturbance decoupling, defect of invariance, differential transformation, invariant operator.**

I. The signal-Parametrical Invariance Problem

The classical Disturbance Decoupling Problem is not complete because this theory is studied only independence of the system response from effect [1]. It limits a class of solved problems and means a noise decoupling in most cases. For the linear systems the problem of parametrical invariance is reduced to a problem of signal invariance, i.e. invariance to external disturbances. A property of invariance of the response parameters or phase coordinates from any effect parameters was not investigated at all. It is an external, signal-parametrical invariance. This kind of Disturbance Decoupling has much more value than invariance in classical sense, and for systems of the most various purposes.

It is known that linear system has property of absolute parametrical invariance. The form of it normalized response does not depends on effect parameters. Response of nonlinear system generally depends on effect parameters. Since in practice the quasi-linear systems are frequently used, it is possible to find an approximation of parameters of nonlinear system to linear. Obviously, it will result in improvement of characteristics because the system, remaining nonlinear, gets another important property of signal-parametrical invariance [2].

Conditions of parameter tolerance of the system response to effect amplitude has paramount value for synthesis. Let’s find these conditions.

II. Condition of Invariance of Overcontrol to the Effect Amplitude

Let's use an idea of paper [3] and describe the equation of the system state as:

$$\sum_{j=1}^{n}(a_{ij}x'_j + b_{ij}x_j) = \begin{cases} Ef(t), & i = 1, \\ 0, & i = \overline{2,n} \end{cases} \tag{1}$$

with response

$$h(t) = \sum_{j=1}^{n} g_j x_j , \tag{2}$$

where x_i are phase coordinates; x'_j are their derivatives on time t; E is amplitude of input effect $f(t)$; n is number of phase coordinates or the system order; a_{ij} are nonlinear functions or coefficients; b_{ij}, g_j are coefficients.

Let's define a condition of invariance of overcontrol σ to amplitude of unit pulse $\sigma = \text{inv}\,[E]$ for the nonlinear system. The system response for the final of transient process is:

$$h_\infty = \sum_{j=1}^{n} g_j x_{j\infty} .$$

For steady state it may be found as a result of solving of (1) at a_{ij}=0. Let

$$\delta_i(t) = x_i(t)/x_{i\infty}(t) - 1 . \tag{3}$$

be the overcontrol function. Substituting (3) in (1), we find:

$$\sum_{j=1}^{n}(A_{ij}\delta'_j + B_{ij}\delta_j) = 0, i = \overline{1,n} \cdot \quad (4)$$

We derive system of equations (4) by *E* and change places for the independent operations ∂/∂*E* and *p*=∂/∂*t*. Finally:

$$\sum_{j=1}^{n}(A_{ij}p + \delta'_j\partial A_{ij} / \partial\delta_j + B_{ij})\partial\delta_j / \partial E = 0, \; i = \overline{1,n} \cdot \quad (5)$$

The system (5) is linear relatively $\partial\delta_j / \partial E$, therefore invariance of overcontrol on a condition $\partial\delta_j / \partial E = 0$ needs equality to zero of the determinant received by deletion of *j*-column and the first line from determinant Δ of system (5):

$$\Delta_{1j} = 0, \quad (6)$$

On the other hand, as system (5) is particular, we have

$$\Delta = 0. \quad (7)$$

Expressions (6) and (7) are simultaneously determined only if determinant lines are proportionality that is not possible. Hence, there exists only a weak invariance of overcontrol to amplitude in nonlinear system.

To evaluate a degree of dependence of the overcontrol from effect amplitude, we claim defect of overcontrol invariance. In this case the relative deviation is more preferable to absolute one because a zero defect will mean a strong invariance:

$$\varepsilon_\sigma = \max_E \left| \frac{\delta(t) - \delta_l(t)}{E\delta_l(t)} \right| \to \min,$$

where $\delta_l(t)$ is overcontrol for the quasi-linear system. Defect of invariance ε gives a quantitative estimation of non- invariance to *E*, therefore (8) is suitable for optimization of systems at invariance criterion. Direct calculation of the invariance defect is connected to decision of (1). However, for practical purposes the estimation (8) is required only that enables to compare systems by criterion of signal-parametrical invariance. Obviously, the purpose of optimization of nonlinear system is reduction of the invariance defect.

It is typical that use of the invariance principle complicates optimization of systems because in practice it is required not only signal-parametrical invariance, but also simultaneous quality of transient.

III. Optimization by Minimum of The Invariance Defect for Overcontrol Function

Let's define an opportunity of transformation of system for conditions of weak overcontrol invariance. We shall describe conditions (4) in form (3):

$$\delta_i' = \theta_i(\delta_j, E),\, i = \overline{1,n},\; j = \overline{1,n}\,, \tag{9}$$

and response of system (2) as $h(t) = \psi(\delta_i, E),\, i = \overline{1,n}$. We derive function ψ on time and replace derivatives of overcontrol functions by the right parts of (9). Then

$$\frac{d^l\psi}{dt^l} = \sum_{i=1}^{n} \frac{\partial}{\partial \delta_i}\left(\frac{d^{l-1}\psi}{dt^{l-1}}\right)\theta_i(\delta_i, E),\, l = \overline{1,n}\,.$$

According to the theorem of invariance [4], the condition of strong invariance of overcontrol is not carried out, because derivatives of functions ψ contain *E*. Also the weak invariance is not achieved at introduction in system (1) the additional signals because it do not influence on amplitude *E* included in (9) as factors of overcontrol functions. Therefore, invariance of overcontrol can be reached by transformation of the system structure or optimization its parameters on criterion of minimum (8).

It is follows from (1) and (8) that searching of $\varsigma = \|[\delta(t) - \delta_l(t)]/[E\delta_l(t)]\|$ maximum is necessary for minimization of invariance defect. The decision of system of equations (1) in a range of input signal amplitudes will demand significant expenses of PC time. Therefore, it is necessary to design algorithm allowing to calculate aim function (8) at value *E* appropriate to supremum ς. Theory of optimum processes [5] is appropriated for its development.

Let's $x_{n+1} = \varepsilon$ be a new variable. With (1) we obviously receive:

$$x_{n+1}' = \frac{d\varepsilon}{dt} = \xi(x, E)\,, \tag{10}$$

where ξ is some function. Suppose that function ξ in $E \in [E_1, E_2]$

has one extremum. Let's T_1 be designated time for the extremum, consequently $\xi(x(T_1), E) = 0$.

Thus, we have a variational problem of searching of amplitude *E* satisfying (1) and (10), that extremum is achieved at moment *T*. A minimum of ε corresponds to strong invariance where $\delta(t) = \delta_l(t)$ in the framework of appropriate amplitude *E*. Maximum corresponds to the most invariance defect.

According to maximum principle [5], *E* is optimum, if there exist function *p*(*t*):

$$\rho_i'(t) = -\sum_{j=1}^{n} \rho_j \partial F_j / \partial x_i,\ i = \overline{1,n};\ \rho(T) = 0, \qquad (11)$$

where F_j is defined from (3) $x_i'(t) = F_i(x_1, x_2, \ldots, x_n, E)$, $i = \overline{1,n}$ and such that for all $T_0 \le t \le T$ amplitude *E* delivers extremum to Hamiltonian:

$$G(x, \rho, E) = \sum_{i=1}^{n} \rho_i F_i - \xi \cdot$$

In case of multi-extremum ε the optimization of system needs definition of admissible area for *E*, that it is possible to integration (1) for boundary of varied parameters. If overcontrol of response is not present, it is possible to use function (3) for optimization.

For example, we consider dependence of overcontrol for the π-low-pass filter with one nonlinear capacity (Fig. 1) from effect. The system of the equation (1) for R_1=R_2=1 Ohm may be received by the theory of circuits:

$$\begin{cases} C_2 x_1' / \sqrt{1 + x_1} + x_1 + C_1 x_2' + x_2 = E; \\ C_2 x_1' / \sqrt{1 + x_1} + x_1 - x_3 = 0; \\ x_1 - x_2 + L x_3' = 0. \end{cases}$$

It was solved by Runge-Kutta-4 method for $E \in [0;8]$, C_2=0.5 F and zero initial conditions. Optimization of C_1, *L* was carried out by reduction of invariance defect (8). Overcontrols calculated for the nonlinear filter and normalized to the appropriate value for the linear circuit are shown in Fig. 2. Diagram C_1=1 F, *L*=l.5 Hn corresponds to initial values, curve C_1=3 F, *L*=2 Hn corresponds to the optimized circuit. As a result of optimization, the invariance defect has decreased in 3.25 times. Strong invariance $\sigma = \sigma_l$ is achieved at parameters C_1=4 F, L=l.7 Hn at region E=2.6 V or C_1=3 F, L=l.5 Hn at region E=5.6 V.

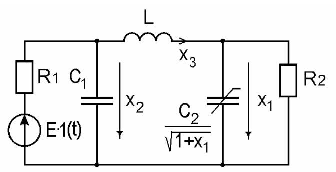

Fig. 1. Low-pass filter with nonlinear capacity

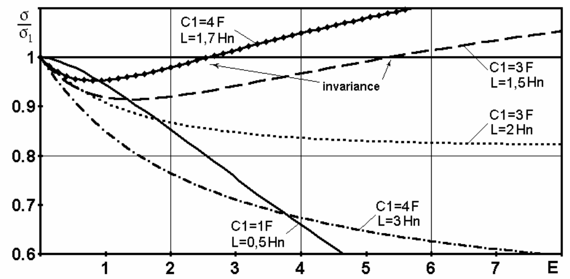

Fig. 2. Overcontrol for nonlinear low-pass filter (Fig. 1) normalized to overcontrol of quasi-linear filter vs effect amplitude

IV. Invariance of Delay for the Pulse Response of Nonlinear System to the Effect Amplitude Defect for Overcontrol Function

Let's define condition of invariance of pulse delay for nonlinear system. For this purpose we shall describe the equation of state of system (1) as:

$$\sum_{j=0}^{\infty} a_j(x) \frac{d^j x}{dt^j} = E1(t), \qquad (12)$$

There exist differential transformation

$$X(k) = \frac{H^k}{k!} \left[\frac{d^k x(t)}{dt^k} \right]_{t=0} \longleftrightarrow x(t) = \sum_{k=0}^{\infty} \left(\frac{t}{H} \right)^k X(k), \qquad (13)$$

where on the left from $\xleftrightarrow{\cdot}$ is transformation of original $x(t) \in \mathbf{R}^n$ into image $X(k) \in \mathbf{R}^n$, on the right is inverse transform *X(k)* in *x(t),* $t \in [0, H]$ is time, *H* is coefficient. *X(k)* is called discrets at specific argument $k \in \mathbf{N}^n$.

In view of properties and table of differential transformation [6], formula (12) will be transformed to spectral model as follows:

$$\sum_{j=0}^{n}\sum_{l=0}^{k} A_j[X(k-l)] \otimes \frac{(l+j)!X(l+j)}{l!H^j} = E\eth(k) \qquad (14)$$

where A_j is the image a_j. If the system is linear, $A_j=a_j$ are constant which in expression (14) are multiplied on appropriate image *X(k).*

Discrets of the time function are consistently defined from the recurrent ratio (14):

$$X(k) = EH^k F_k(A_0, A_1, ..., A_n, E)/k!, \qquad (15)$$

where F_k is some functions independent from *E* if the system is linear. According to (13), from (15) we find:

$$x(t) = \sum_{k=0}^{\infty} Et^k F_k(A_0, A_1, ..., A_n, E)/k!.$$

Time of pulse delay is defined on a level $x(t_z) = 0{,}5Eh_\infty$:

$$0{,}5h_\infty = \sum_{k=0}^{\infty} t_z{}^k F_k(A_0, A_1, ..., A_n, E)/k!, \qquad (16)$$

From (16) follows that if the system is linear, delay time does not depends on amplitude *E*. In nonlinear system the function F_k depends from *E*, hence, in nonlinear system only weak invariance is possible [7].

Condition of invariance directly follows from (15). In order to invariance functions F_k should not contain *E*:

$$F_k[X(k), H, E] = k!X(k)/(EH^k) = F_k[X(k), H]. \qquad (17)$$

By analogy it is possible to show that fulfillment of (17) is necessary for any temporary parameters of the pulse response of system, for example, for rise-time $t_f = t_2 - t_1$, where t_1 and t_2 is time points where *x(t)* equally $0{,}1Eh_\infty$ and $0{,}9Eh_\infty$ accordingly. The condition (17) is necessary, but insufficient because functions F_k in a weak nonlinearity of system do not depend from *E*, but only weak invariance will be achieved in system.

Reduction of invariance defect can be achieved by transformation of system structure or change of its parameters. For this purpose the condition of invariance (17) may be used. If differential equation of nonlinear system or its separate plants is known, the decision frequently can be received in an analytical kind. Especially this method is effective for systems with strong nonlinearity.

For example, we shall consider a consecutive oscillatory *RLC*-circuit with nonlinearity $C=C_0/x$, where x is voltage on capacity. Let's the differential equation of circuit

$$\frac{d^2x(t)}{dt^2}+\frac{R}{L}\frac{dx(t)}{dt}+\frac{x^2(t)}{LC_0}=\frac{x(t)}{LC_0}E1(t); \qquad (18)$$

$$x(0)=dx(0)/dt=E$$

describe in the differential-spectrum form:

$$LC_0(k+1)(k+2)X(k+2)/H^2+RC_0(k+1)X(k+1)/H+ \\ +\sum_{l=0}^{k}X(k-l)X(l)=EX(k),$$

$$X(0)=E,\ X(1)=EH.$$

Substituting consistently k=0,1,2., we find discrets $X(2)$, $X(3)$... Similarly we find discrets of quasi-linear equations:

$$\frac{d^2x_l(t)}{dt^2}+\frac{R}{L}\frac{dx_l(t)}{dt}+\frac{x_l(t)}{LC_0}=\frac{E1(t)}{LC_0},$$

$$LC_0(k+1)(k+2)X_l(k+2)/H^2+ \\ +RC_0(k+1)X_l(k+1)/H+X_l(k)=Eъ(k).$$

First discrets for both equations are presented in Table.

From Table, functions $F_k=k!X(k)/(EH^k)$ of the quasi-linear equation (17) do not depend from E that proves its invariance. Functions F_k of the initial nonlinear equation (18) depend from E. Taking into account that decision from discrets is restored by (13), for the invariance conditions the first discrets is more valuable because they bring the biggest contribution to a total amount. For example, at k=3 it is received $R^2C_0=EL$ (weak invariance), at k=3 we have equality $R=L$ etc.

From Table one more general conclusion is followed: discrets of decision of the nonlinear equation consist from discrets of the quasi-linear equations plus discrets from defect of invariance. Hence, the mathematical model of system may be submitted as decomposition on parameter-invariant system and defect of invariance.

Table. Discrets of nonlinear and quasi-linear equations for a *RLC*-circuit

k	$X(k)$	$X_l(k)$
0	E	E
1	EH	EH
2	$-\dfrac{REH^2}{2!L}$	$-\dfrac{REH^2}{2!L}$
3	$\dfrac{EH^3}{3!LC_0}\left(\dfrac{R^2C_0}{L}-E\right)$	$\dfrac{EH^3}{3!LC_0}\left(\dfrac{R^2C_0}{L}-1\right)$
4	$-\dfrac{EH^4}{4!LC_0}\left(\dfrac{R^3C_0}{L^2}-\dfrac{2RE}{L}+2E\right)$	$-\dfrac{EH^4}{4!LC_0}\left(\dfrac{R^3C_0}{L^2}-\dfrac{2R}{L}\right)$
5	$\dfrac{EH^5}{5!L^2C_0}\left(\dfrac{R^4C_0}{L^2}-\dfrac{3R^2E}{L}+\dfrac{E^2}{C_0}+8RE\right)$	$\dfrac{EH^5}{5!L^2C_0}\left(\dfrac{R^4C_0}{L^2}-\dfrac{3R^2}{L}+\dfrac{1}{C_0}\right)$

It is possible to receive the decision of the delay time invariance problem for weak nonlinearity systems by optimization. Objective function represents defect of invariance:

$$\varepsilon_t = \max_E \left|\frac{t_z - t_{zl}}{t_{zl}}\right| \to \min,$$

where t_z and t_{zl} is delay time of responses of nonlinear system (12) and hypothetical linear system accordingly. It is expedient to use system (12) under condition of $a_j = const$, that following to linearization of system.

In order to take advantage of the developed algorithm, we use new variable $x_{n+1} = t$ and we shall add to system (1) one more equation

$$x'_{n+1} = 1 \qquad (20)$$

with initial condition $x_{n+1}(0) = T_0$. We have a variational problem of search amplitude E satisfying (3) and (20), that at the time T_1 the extremum of invariance defect (19) was achieved. According to a maximum principle [5], E is optimum if there exists extremum of functional

$$G(x,\rho,E)=\sum_{i=1}^{n}\rho_i(t)F_i(x,E,x_{n+1})+\rho_{n+1}(t),$$

where for conjugate variables $\rho_i(t)$ equality (11) are carried out and

$$\rho'_{n+1}(t)=-\partial G/\partial t\,.$$

V. Optimization of System by the Minimum Invariance Defect

Complexity of the numerical-analytical approach to the decision of signal-parametrical invariance problem demands development of numerical methods of optimization by criterion of the invariance defect minimum. One of approaches is the use of the formula for the functional variation [7].

Let there exist two phase trajectories of system (1). The first *x(t)* corresponds to initial conditions $x(T_0)=x_0$ and amplitude of signal E, the second $x_n(t)$ corresponds to initial conditions $x_{n0}=x_0+\Delta x_0$, $T_{n0}=T_0+\Delta T_0$, $E_n=E+\Delta E$. Then the difference between values of function (2) for these trajectories may be calculated by [7, 8]:

$$\Delta h=h(x_n,T_n)-h(x,T)=-(\rho(T_0),\Delta x_0)+G(x,\rho,Ef(t),T_0)\Delta T_0-\\-\int_{T_0+\Delta T_0}^{T+\Delta T}\left[G(x,\rho,(E+\Delta E)f(t),t)-G(x,\rho,Ef(t),t)\right]dt \qquad (21)$$

Here G is Hamiltonian, ρ is a vector, determined along trajectory *x(t)* as the decision of:

$$\rho'_i(t)=-\frac{\partial G}{\partial x_i}-\sum_{j=1}^{n}\rho_j\frac{\partial F_j}{\partial x_i},\ i=\overline{1,n};\ \rho(T)=0\,; \qquad (22)$$

$$G(x,\rho,Ef(t),t)=(\rho,F(x,Ef(t),t))=\sum_{i=1}^{n}\rho_iF_i\,. \qquad (23)$$

Ratio (21) gives an opportunity to write down the following condition of invariance based on the Rosonoer theorem [4]:

$$G(x,\rho,Ef(t),t)=G(x,\rho,f(t),t)=\mathrm{inv}[E]\,. \qquad (24)$$

In turn, it gives to develop of optimization algorithm for nonlinear system by criterion of phase coordinate invariance to amplitude of signal E.

There exist new variable in system (3)

$$x_{n+1} = \varepsilon = \left| h(t) - h_l(t) \right| \quad (25)$$

with the initial condition $x_{n+1}(0) = T_0$. Here $h_l(t)$ is the response of hypothetical linear system. It is the system (1) with a_{ij}=const that results to linearization of (1). From (24) follows:

$$x'_{n+1} = d\varepsilon / dt = \xi(x, E), \quad (26)$$

where $\xi(x, E)$ is some function. We shall consider that at $E \in [E_1, E_2]$ function ξ has one extremum. There exist $\xi(x(T_1), E) = 0$ where T_1 is extreme point.

Thus, we have a variational problem of search the amplitude E satisfying the equations (1) and (26) that at moment T_1 the extremum of (25) was achieved. According to a maximum principle [5], value E was optimum if it exist such conjugate ρ for function $T_0 \le t \le T_1$ (22) that for all amplitude E delivers an extreme of Hamiltonian (23). In case of a minimum the point of strong invariance and appropriate amplitude will be found where $h(t) = h_l(t)$. In case of a maximum the largest defect of invariance will be found. In case of multi-extremeness ε the optimization of system needs definition of region of admissible E that it is possible to make integration of system (1) for boundary values of varied parameters.

For low-pass filter from previous example the minimization of defect (19) results curves in Fig. 3. As well as in a case with over-control (Fig. 2), diagram C_1=1 F, L=l.5 Hn corresponds to initial conditions. Result of optimization of the filter parameters (C_1=4 F, L=3 Hn) are received. Strong invariance where $t_z = t_{zl}$ is achieved only in case of a weak signal.

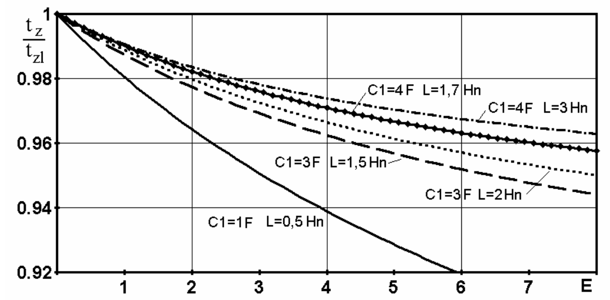

Fig. 3. Dependence of the pulse delay time in the nonlinear low-pass filter (Fig. 1) normalized to the pulse delay time in the linearization filter from amplitude *E*

VI. Invariance and Asymptotic Equivalence

From strong invariance condition the comparison of nonlinear system with some hypothetical linear system with absolute signal-parametrical invariance is necessary. We shall consider system of equations

$$dx / dt = Ax + Bu \,, \quad (27)$$

in which A is *(nxn)* matrix. Obviously, the given system is parametric invariance. We shall describe a system (27) in the operational form:

$$\frac{k+1}{H} X(k+1) = AX(k) + BU(k) \cdot \quad (28)$$

Assume that there exist some nonlinear system

$$d\widetilde{x} / dt = A\widetilde{x} + Bu + f(t,\widetilde{x}) \,, \quad (29)$$

where $f : D \to R^n$, $D = \{(t,\widetilde{x}) : \widetilde{x} \in R^n\}$ for which follows that $x = \widetilde{x}$ from $f(t,\widetilde{x}) = 0$. In the operational form:

$$\frac{k+1}{H} \widetilde{X}(k+1) = A\widetilde{X}(k) + BU(k) + F(k) \cdot \quad (30)$$

From (28) and (30) follows that $\widetilde{X}(k) \to X(k)$ at $F(k) \to \min$. In this case $x(t,u) = \widetilde{x}(t,u) + o[x(t,u)]$, if $\|f(t,y)\| \le \lambda(t,\|y\|)$, $\lambda(t,\alpha_1) \le \lambda(t,\alpha_2)$ at $\alpha_1 \le \alpha_2$ and $\forall t \ge t_0$. It means that systems (27) and (29) are equivalent on Brauer [9]. In [1] it is shown that if $x(t)$ is decision of any another asymptotic nonequivalent system of kind

(29), defect of invariance will be more. At any nonlinear functions the invariance defect can not be less than some limiting value in asymptotic nonequivalent systems. Meanwhile, realization of parametrical invariance conditions, even not absolute, but relative, improves quality of systems and allows to design systems with new properties.

VII. Conclusions

In the paper, the property of invariance for parameters of the of pulse response of nonlinear system to signal parameters on example of invariance to effect amplitude is considered. Conditions for defect of invariance are received and its reduction is considered. Defect of invariance is a quantitative estimation as far as the target signal is dependent to amplitude. It is shown an existence of amplitudes of input signals which zero defect of invariance is achieved. The searching algorithm of the amplitude and optimization of parameters of nonlinear system by criterion of minimum of invariance defect is offered.

The mathematical model of nonlinear system may be obtained as decomposition on parametrical invariant system and invariance defect. The purpose of designing is the choice parametrical invariant system and reduction of invariance defect.

An initial conditions and signal amplitude includes in formulations of invariance conditions of the nonlinear system. Conditions of invariance are the guarantee of invariance only in the certain time points. However it is necessary to have an invariant property in a wide amplitude range for continuous objects and processes.

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