Use of Differential-Taylor Transformation in Problems of Characterization of the Nonlinear Amplifier

Oleg V. Stukach
TPU, 30 Lenin Avenue, Tomsk, 634050, Russia, email: ieee@main.tusur.ru

Abstract – In this paper, the Differential-Taylor Transformation are considered as the operator method, allowing to characterization of the nonlinear devices. Advantages of the method are shown from simple example. The aim is to give an insight into Differential-Taylor Transformations as the power method with its advantages – from analytical simulations to numerical solution.

Index Terms – Differential-Taylor Transformation, image, original, Taylor series, nonlinear circuit

I. INTRODUCTION

The operational methods based on integral transformations have shown for many years high efficiency for the system studying, which state is described by ordinary differential equations. However, from papers on the integral methods application for studying the periodic processes in multivariable systems with nonlinear and variable parameters is seen that decision of arising problems is possible only in quasi-linear mode. More possibility for studying of the nonlinear system has Differential-Taylor Transformation (DTT) [1].

The DTT advantage is possibility of distribution of operating methods on systems with variable and nonlinear parameters. The main difference of DTT from integral transformation consists in follows: transition from original to image is carried out by differentiation of original. Essence of method consists in transformation of the original function from continuous argument, for instance, time, in the image function of discrete argument, which coefficients are called discrets. Rules and formulas in the DTT theory allow in practice formation the representing equations without differentiation of originals. The inverse transition from the images to the originals performs by Taylor series very simply.

The primary aim of this paper is to show a possibility of DTT using for characterization and parametrical synthesis of the nonlinear devices on different criterions in time domain. For the purpose of DTT illustrating, we restrict the range of questions only by characterization of the transistor amplifier. More information about DTT theory is provided in papers [1–4].

II. SHORT INTRODUCTION TO THE DIFFERENTIAL-TAYLOR TRANSFORMATION THEORY

Direct and inverse functional Differential Transformation is possible to describe as following [1–2]:

\[ X(k) = \frac{H^k}{k!} \left[ \frac{d^k x(t)}{dt^k} \right]_{t=0} \equiv x(t) = \sum_{k=0}^{\infty} \left( \frac{t}{H} \right)^k X(k), \quad (1) \]

where on the left is the direct transformation of the original \( x(t) \in \mathbb{R}^n \) as function of continuous time \( t = [0, H] \) in the image \( X(k) \in \mathbb{R}^n \) as function of discrete argument \( k = 0, 1, 2, \ldots, \infty \), and from the right in the inverse transformation \( X(k) \) in \( x(t) \); \( H \) is normalized coefficient. Values of function \( X(k) \) at specific argument \( k \in \mathbb{N}^n \) are called discrets: \( X(0) \) is zero discret, \( X(1) \) is the first discret etc.

From expression (1) follows that function \( x(t) \) and its derives in a point \( t = 0 \) in the operator form will be transformed in discrete spectrum. In turn, discret enables to define the function \( x(t) \) in a form of extent Taylor series.

Here is simple example. Let the original be \( x(t) = e^{-t} \). Then image function is

\[ X(k) = \frac{H^k}{k!} \left[ \frac{d^k e^{-t}}{dt^k} \right]_{t=0} = (-H)^k \left[ e^{-t} \right]_{t=0} = (-H)^k \cdot \frac{H^k}{k!}. \]

Fig. 1. DT-spectrum for exponent at \( H=9 \)

Spectrum of the infinitely differentiated functions is infinited. It is obvious that DT-spectrum of polynomials is always limited, since polynomial has finite number of nonzero derivations.
III. EXAMPLE OF THE TABLE OF DIFFERENTIAL TRANSFORMATION

The images of the elementary functions are obtained by means of direct DTT (1), and more complex one – by representation of the originals via equivalent differential equations. In most cases, for the transform of the initial equations in the image domain, it is sufficient to use the table [1], the small part of which one is given below.

<table>
<thead>
<tr>
<th>Original</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x(t) = \sum_{n=0}^{\infty} \left( \frac{t}{H} \right)^n X(k)$</td>
<td>$X(k) = \frac{H^n}{k!} \left[ \frac{d^n x(t)}{dt^n} \right]_{t=0}$</td>
</tr>
<tr>
<td>$e^{ct}$</td>
<td>$\left( \frac{cH}{k!} \right)^n \delta(k)$</td>
</tr>
<tr>
<td>$\sin(\omega t + \alpha)$</td>
<td>$\left( \frac{\omega H}{k!} \right)^n \sin \frac{\pi k}{2} + \alpha$</td>
</tr>
<tr>
<td>$\cos(\omega t + \alpha)$</td>
<td>$\left( \frac{\omega H}{k!} \right)^n \cos \frac{\pi k}{2} + \alpha$</td>
</tr>
<tr>
<td>$\ln(1 + ct)$</td>
<td>$\left( \frac{cH}{k} \right)^n (\delta(k) - \cos(\pi k))$</td>
</tr>
<tr>
<td>$e^{-ct} \sin(\omega t)$</td>
<td>$\left( \frac{\omega H}{k!} \right)^n \sum_{i=0}^{n-1} \left( \frac{c}{\omega} \right)^i \sin \frac{\pi i}{2}$</td>
</tr>
<tr>
<td>$e^{-ct} \cos(\omega t)$</td>
<td>$\left( \frac{\omega H}{k!} \right)^n \sum_{i=0}^{n-1} \left( \frac{c}{\omega} \right)^i \cos \frac{\pi i}{2}$</td>
</tr>
<tr>
<td>$x(0)$</td>
<td>$X(0) \delta(k)$</td>
</tr>
<tr>
<td>$ax(t) \pm by(t)$</td>
<td>$aX(k) \pm bY(k)$</td>
</tr>
<tr>
<td>$x(ct)$</td>
<td>$e^{c^2} X(k)$</td>
</tr>
<tr>
<td>$x(iy(t))$</td>
<td>$X(k) \otimes Y(l) = \sum_{i=0}^{l} X(k - i) Y(l)$</td>
</tr>
<tr>
<td>$\frac{d^n x(t)}{dt^n}$</td>
<td>$\left( \frac{k + m}{k!} \right)^n X(k + m)$</td>
</tr>
<tr>
<td>$\int x(t) dt$</td>
<td>$H \int \frac{X(k - 1)}{k} + C \delta(k)$</td>
</tr>
<tr>
<td>$\frac{dx(t)}{dt} + Ax(t) = f(t)$</td>
<td>$(k + 1)X(k + 1) / H + AX(k) = F(k)$</td>
</tr>
</tbody>
</table>

Thereby, the given mathematical model of object in

the form of system of the integral-differential equations will be transformed in so-called spectral model by replacement of functions and operations by differential spectrums and usual operations on functions – by appropriate operations on differential spectrums. DTT allows to transform in the algebraic form the initial differential equations of any kind by practically one way.

IV. PARAMETRICAL SYNTHESIS BY INVERSE DTT

Let structure of an object in a kind of the ordinary differential equation of n order be:

$$A_n \frac{d^n x(t)}{dt^n} + A_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + \ldots + A_1 \frac{d x(t)}{dt} + A_0 x(t) = E(t)$$

where $A_i$ (i=0,...,n) are constants, $E(t)$ is external impact: $E(t) = \{0, \quad t \leq 0 \}$

The transitive pulse characteristic $p(t)$, on the interval $t=[t_0,\ldots,t_0]$, is given by tabulated, or graphic way. Some parameters of the equation (2) $A_i$ are unknown. It is necessary to define unknown parameters on the given characteristic $p(t)$.

One of the important features of the DTT theory is that the spectral model represents itself a system of recurrence equations, from which the discrete of differential spectrum of the required values are easy founded. Using the table of DT-Transformations, we shall transform the initial equation (2) in the image domain and receive the recurrence expression for calculation of DT-spectrum:

$$X(k + n) = \frac{k! H^n}{A_n (k + n)!} \left[ E \delta(k) - \sum_{i=0}^{n-1} \left( \frac{A_i X(k + i)}{k! H^n} \right)^{k+i} \right]_k, \quad k = 0, 1, 2, \ldots$$

Consistently substituting in expression (3) $k=0, 1, 2, \ldots n-1$, we shall receive $n$ linear independent equations:

$$X(k + n) = \frac{H^n}{A_n (k + n)!} \left[ E \sum_{i=0}^{n-1} \left( \frac{A_i X(k + i)}{H} \right)^{k+i} \right]_k, \quad k = 0, 1, 2, \ldots n-1$$

Solving received system of the equations (4) at known $X(m)$, $m=0, 1, 2, \ldots 2n-1$, it is possible to find of $n$ coefficients $A_i, i=0, \ldots, n$. For the small order of system (2) it is simply to find the analytical expression for coefficients $A_i$ vs $X(m)$.

For calculation of $X(m)$ we shall take on the given diagram of the transitive pulse characteristic $p(t)$ inside an interval $[t_0, \ldots, t_0]$ $N+1$ points ($N>2n$) and describe for each of them the equation, using the inverse transformation (1):
\[ p(t_i) = \sum_{k=0}^{N} \left( \frac{f(t_i)}{H} \right)^k \times X(k). \tag{5} \]

At \( H = 1 \) we receive the system of linear algebraic equation:
\[ p(t_i) = \sum_{m=0}^{N} X(i) t_i^m, \quad i = 0, N \tag{6} \]

where \( t_i \) are the temporary points inside the interval \([t_0, t_n]\). Solving system (6), we shall receive discreet \( X(m) \) and calculate required coefficients \( A_i \) (4).

In a case of strongly nonlinear system there will be the discrete convolution as \( X(k) \otimes Y(k) \) in expression (3), and the recurrence system (4) will be also nonlinear. Solving the system and restoring the first parts of the series (1), we receive conditions for engineering design of the nonlinear device.

V. EXAMPLE OF THE ELECTRICAL CIRCUIT

For example, we shall calculate the transitive characteristic of an amplifier on the bipolar transistor KT610A in the common emitter circuit (Fig. 2).

\[ U_{K} = E - I_{K} \cdot R_{K} = 2.77 \text{ V}. \tag{8} \]

Let us describe the system of the differential equations for considered circuits (Fig. 3):
\[ E = I_{K}(R_{K} + \Delta r) + U_{C_{K}} + U_{C_{E}}, \tag{9} \]
\[ E_{C} = I_{C}(R_{g} + r_{b} + \Delta r) + U_{C_{E}}, \]
\[ E = I_{C}(R_{g} + r_{b} + \Delta r) + U_{C_{E}}, \]
\[ I_{K} = C_{k} \frac{dU_{C_{K}}(t)}{dt} + h_{21e}(I_{E} + I_{3}), \]
\[ I_{K} + I_{C} + I_{3} = C_{e} \frac{dU_{C_{E}}(t)}{dt} + U_{C_{E}}. \]

Reduce system (9) to one equation concerning the collector voltage at the input signal:
\[ A_{1} \frac{dU_{C_{K}}(t)}{dt} + A_{2} \frac{dU_{C_{E}}(t)}{dt} + A_{3} U_{C_{E}}(t) = A_{4}, \tag{10} \]

where
\[ A_{1} = C_{k}C_{e}, \quad A_{2} = C_{k} + C_{e} \frac{R_{K}}{R_{1}}, \quad A_{3} = A + y, \quad A_{4} = \frac{1}{R_{2}} - \frac{h_{21e}y}{R_{2}}, \]
\[ A_{4} = E \left( \frac{1}{R_{3}} + \frac{1}{R_{4}} + \frac{h_{21e}y}{R_{2}} + \frac{y}{R_{2}} \right) + E_{C} \left( \frac{1}{R_{2}} - \frac{h_{21e}y}{R_{2}} \right). \]
\[ R_{1} = R_{K} + \Delta r = 115.7 \text{ Ohm}, \]
\[ R_{2} = R_{g} + r_{b} + \Delta r = 8016 \text{ Ohm}, \]
\[ R_{3} = R_{b} + r_{b} + \Delta r = 40 \text{ kOhm}, \quad R_{4} = r_{e} = 0.256 \text{ Ohm}, \]
\[ A = \frac{h_{21e}}{R_{2}} + \frac{h_{21e}}{R_{2}} - \frac{1}{R_{3}} = 9.697 \times 10^{-3} \text{ Ohm}^{-1}, \]
\[ y = \frac{1}{R_{3}} + \frac{1}{R_{4}} + \frac{1}{R_{1}} = 3.915 \text{ Ohm}^{-1}. \]

Use the table 1 of DT-transformations we shall transform (10) into the image domain:
\[ A_{1} \frac{X(k+2)(k+2)}{H^{2}k!} + A_{2} \frac{X(k+1)(k+1)}{H} + A_{3} X(k) = A_{4}X(k), \tag{11} \]

then we shall received \( X(k+2) \) and following recurrence ratio for discreet of voltage on the collector capacity:
\[ X(k+2) = H^{-1}A_{4}X(k) - A_{3}X(k), \quad \frac{H A_{4}X(k+1)}{A_{3}(k+2)}. \]

where \( X(0) = U_{C_{E}}(0) \) and \( X(1) = 0 \) are calculated initial conditions. From (11) we have:

![Fig. 2. Amplifier on the bipolar transistor](image)

![Fig. 3. The equivalent circuit of stage (Fig. 2)](image)


\[ X(2) = \frac{(A_4 - A_2 U_{C_0}) H^2}{2 A_1} \]

\[ X(3) = -\frac{A_2 (A_4 - A_2 U_{C_0}) H^2}{6 A_1^2}, \quad \ldots \quad (12) \]

Likewise expression for discrets on the emitter capacity voltage is:

\[ B_1 \frac{d^2 U_{C_0}}{dt^2} + B_2 \frac{dU_{C_0}}{dt} + B_3 U_{C_0}(t) = B_4, \]

where \( B_1 = C_e C_K R \), \( B_2 = C_e + C_K y R \),

\[ B_3 = A + y, \quad B_4 = \frac{E + E_e}{R_3 + R_4} \left( b_{1(k)} + 1 \right) \]

Recurrence formula for calculation of discrets of voltage on the emitter capacity is:

\[ Y(k+2) = \frac{H^2 [B_3 \delta(k) - B_2 Y(k)] - H B_2 Y(k+1)}{B_3 (k+1) (k+2) - B_4 (k+2)}, \quad (13) \]

where \( Y(0) = U_{C_0}, Y(1) = 0 \) are initial conditions.

Substituting calculated discrets on (12) and (13) in (1), we shall receive the required decisions in a kind of Taylor series (1).

Recurrence formula for calculation of discrets of voltage on the collector capacity is:

\[ U_{C_0}(t) = U_{C_0}(k) + \frac{H^2}{B_3} \left( \frac{B_2}{B_3} U_{C_0}(k+1) - B_2 U_{C_0}(k) \right) \]

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Recurrence formula for calculation of discrets of voltage on the emitter capacity is:

\[ B_1 \frac{d^2 U_e}{dt^2} + B_2 \frac{dU_e}{dt} + B_3 U_e(t) = B_4, \]

where \( B_1 = C_e C_K R \), \( B_2 = C_e + C_K y R \),

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