

# Characterization of the Nonlinear FET Model in the Mode of Amplitude Pulse Invariance

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**Abstract** – A problem of stabilization of the pulse risetime depending amplitude in nonlinear FET amplifiers is considered. Modelling of the amplifier sections is carried out. It is established that nonlinear properties of FET results to dependence of the risetime from effect amplitude. The computing minimization of the temporary parameters independence from pulse effect amplitude has been proposed. The aim is to give an insight into Differential-Taylor Transformations as the power method with its advantages.

**Index Terms** – Differential-Taylor Transformation, image, original, Taylor series, pulse invariance, nonlinear circuit

## I. Introduction

Recently the new computing methods for the parametrical synthesis of the nonlinear devices are arisen. However the theoretical boundaries in achievement of the limited characteristics are not achieved yet [1]. For example, it is the problem of the response invariance to the effect amplitude. This problem is not solving in the classical basis since the operator functions has high order.

Researches on using the integral methods for studying the periodic processes in multivariable systems with nonlinear and variable parameters described that decision of arising problems practically turns out to be possible only in quasi-linear mode. More possibility for studying of the nonlinear system has Differential-Taylor Transformation (DTT) [1–2].

In this paper the Differential-Taylor Transformations are considered as the operating method, allowing to carry out a characterization of the nonlinear devices.

## II. Condition of the risetime invariance

A main way of retain of the signal form in the amplitudes range is linearization [3]. Decision of this problem is suggest an invariance of the target form to the effect amplitude. Therefore linearization of pulse systems can be based on approximation of the separate parameters of the

nonlinear system response to the form of the response of some ideal linear system.

Let us determine the invariance condition of the pulse risetime in the nonlinear system. The equation of the system we describe in a kind:

$$\sum_{j=0}^{\infty} a_j(x) \frac{d^j x}{dt^j} = E1(t), \quad (1)$$

where  $x \in D(t)$  is response of system;  $E \in U(t)$  is source signal amplitude;  $t$  is continuous time;  $n$  is system order;  $a_j(x)$  are the nonlinear functions or coefficients for nonlinear system or coefficients for linear systems;  $D, U$  are some manifolds. Using the differential transformation [2]

$$X(k) = \frac{H^k}{k!} \left[ \frac{d^k x(t)}{dt^k} \right]_{t=0} \equiv x(t) = \sum_{k=0}^{\infty} \left( \frac{t}{H} \right)^k X(k), \quad (2)$$

where  $X(k)$  is discrets of a differential spectrum (image) of system variables  $x(t)$ ;  $k$  is discrete argument;  $H$  is constant, equation (1) is written as:

$$\sum_{j=0}^n \sum_{l=0}^k A_j [X(k-l)] * \frac{(l+j)! X(l+j)}{l! H^j} = \begin{cases} E, k=0; \\ 0, k>1, \end{cases} \quad (3)$$

where  $A_j$  is image of  $a_j$ , and symbol \* means an operation of discrete convolution

$$X(k)Y(k) = \sum_{l=0}^k X(k-l)Y(l).$$

Thereby, the given mathematical model in a kind of the integral-differential equations will be transformed in so-called spectral model.

If the system is linear,  $A_j = a_j$  are constants, which in expression (3) are multiplied on the reciprocal image  $X(k)$ . The equation (3) represents itself the recurrence expression, from which the discrete of temporary function  $X(k)$  with the initial conditions are consistently determined:

$$X(k) = EH^k F_k(A_0, A_1, \dots, A_n, E) / k!, \quad (4)$$

where  $F_k$  are some functions not dependent from  $E$ , provided that the system is linear. According to (4) we find:

$$x(t) = \sum_{k=0}^{\infty} Et^k F_k(A_0, A_1, \dots, A_n, E) / k!.$$

The risetime  $t_z$  is determined on the  $x(t)=0.5E$  level:

$$0.5 = \sum_{k=0}^{\infty} t_z^k F_k(A_0, A_1, \dots, A_n, E) / k!. \quad (5)$$

It follows from this expression that risetime does not depend on effect if the system is linear. In the nonlinear system the function  $F$  depends on the effect  $E$ , hence, in nonlinear systems defect of invariance is inevitable, that is deviation of characteristics of the nonlinear system from linear. So we receive a condition of invariance:

$$F_k [X(k), H, E] = k! X(k) / (EH^k). \quad (6)$$

### III. The FET amplifier

The equivalent circuit of the picosecond nonlinear pulse amplifier is shown on fig. 1. It has small dependence of risetime of pulse response from the effect amplitude owing to matching circuit  $L_k C_k$ . Parameters of the equivalent circuit are following:  $R_1=5$  Ohm;  $R_2=1$  Ohm;  $C_2=0.07$  pF;  $C_1=0.3/\sqrt{1-x}$  pF;  $R_n=50$ ;  $C_k=0.1$  pF.

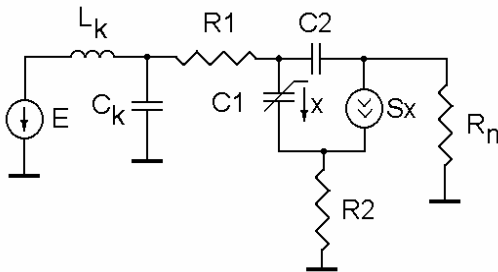


Fig. 1. The amplifier equivalent circuit

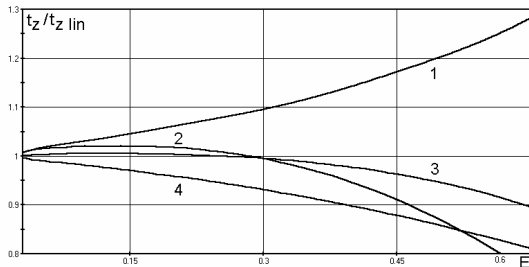


Fig. 2. Risetime vs. effect amplitude for different  $L_k$  (1 –  $L_k=1$  pH; 2 – 0.01 nH; 3 – 0.1 nH; 4 – 0.2 nH)

As a result of the circuit optimization the influence of nonlinear  $C_1$  to the pulse characteristic is reduced. The normalized characteristics of dependence of risetime from effect amplitude for investigation sections are shown on the fig. 2. Optimized parameter was  $L_k$ . From fig. 2 is visible that  $L_k=0.1$  nH is optimal on criterion of the risetime invariance. Maximum of dependence  $t_z$  vs.  $E$  is reduced in 3 times approximately. Risetime did not exceed 82 ps in the most effect amplitude, when influence of nonlinearity's is maximal value.

Discrets of the output voltage normalized to  $EH^k/k!$  vs.  $L_k$  at the invariance point  $E=0.3$  V has minimum. It proven that condition (6) may be used for optimization of the nonlinear system by invariance criterion.

### IV. Conclusion

In the paper, the characteristics of model of the picosecond amplifiers developed in the Picosecond Technique Labs of Tomsk Polytechnic University are presented. The picosecond device optimization technique is offered, which permits partially to solve one of the problems of the risetime response stability in the pulse amplifiers. Basing on the theoretical researches the amplifiers with 20 dB gain and frequency band from zero up to 11.3 GHz, rise-time from 40 ps up to 10 ns; dynamic range of 90 dB are developed by the hybrid thin-film technology.

The problem of invariance of the pulse risetime from effect amplitude in nonlinear amplifiers is considered. Basing of the differential transformation the condition of invariance is determined. Modeling of the amplifier sections is carried out. Such devices are applicable in subsurface radar, oscillography, in broadcast systems, including fiber-optic, experimental physics, etc.

### References

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