Exactitude of the Electronic Devices Analysis by the Differential Transformations Method

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Abstract—**The paper presents a new method of the electronic devices analysis by the differential transformations method. This method allows to research as linear devices as nonlinear and with variable parameters. The problem of poor accuracy at great time interval is discussed and way of accuracy increase is offered.**

I. Introduction

EVERYBODY knows Laplace's transformation. This integral transformation helps to transform the initial differential equations of an object condition to more simple algebraic equations. Advantages of the method – an opportunity of exact calculation of integral, i.e. exact enough numerical methods. However their application to the analysis of systems with variable and nonlinear parameters, especially at studying non-stationary processes, does not give appreciable advantages in comparison with known numerical methods.

There are differential transformations along with it. The original is transforming to image by the help of differentiation operation, and return conversion is carried out as Taylor's series. Transformation looks as shown in the formula (1).

$$X(k)=\frac{H^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \quad \equiv \quad x(t)=\sum_{k=0}^{\infty}\left(\frac{t}{H}\right)^k X(k) \qquad (1)$$

where at the left there is a direct transformation of original $x(t)$ into image $X(k)$, and on the right – reconversion $X(k)$ to $x(t)$. Values of the function $X(k)$ at concrete values of argument k are named discrets ($X(0)$ – zero discrete, $X(1)$ – first discrete, etc.).

As well as Laplace's transformation the given method has the table of originals translation into images too [1]. Advantage of these transformations that simply enough both the linear equations and nonlinear and with variable parameters are translated in area of images. Essential lack consists in difficult calculations. To receive exact enough solution, it is necessary to have a lot of discrets, i.e. to calculate high degrees and factorials. Therefore until recently these transformations have not got a wide spreading.

Now because of development of computer equipment and software problems of calculations became more solvable.

II. The Problem

The basic advantage of differential transformations is that after translating the initial integral-differential equations in area of images the recurrent formula for discrets calculation turns out. Knowing the first discrete value, which is determined by entry conditions, it is possible to calculate all others discrets. After that, using retransformation (1), we have the decision as power series.

Let's show an example of calculation of the transitive characteristic of an integrating circuit with nonlinear capacity:

The capacity depends on the voltage on it as follows:

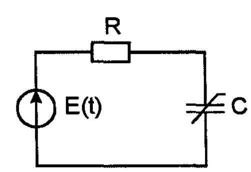

Fig. 1. The Integrating circuit with nonlinear.

$$C=C_0\cdot\frac{U_0}{b\cdot U(t)+U_1}, \qquad (2)$$

$U(t)$ is the voltage on the capacitor, U_0 and U_1 are some constants, $U_0=U_1=1$, b is nonlinearity factor, at b=0, it will be the linear circuit.

Write down the differential equation for the given circuit:

$$\frac{d}{dt}U(t)=\left(E(t)-U(t)\right)\cdot\frac{b\cdot U(t)+U_1}{R\cdot C_0\cdot U_0}, \qquad (3)$$

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where

$$E(t) = \begin{cases} 0, t < 0 \\ E_{\max}, t >= 0 \end{cases}$$

Having taken advantage of the table of transformations [1], convert (2) in area of images and take the recurrent formula for discrets calculation:

$$X_{k+1} = \frac{H}{k+1}\left(\rho(k)\cdot\frac{E_{max}\cdot U_1}{R\cdot C_0\cdot U_0} + X_k\cdot\frac{b\cdot E_{max} - U_1}{R\cdot C_0\cdot U_0} - \frac{b}{R\cdot C_0\cdot U_0}\cdot\sum_{l=0}^{k} X_{k-l}\cdot X_l\right) \quad (4)$$

where

$$\rho(k) = \begin{cases} 1, k = 0 \\ 0, k \neq 0 \end{cases}$$

$X(k)$ – the T-spectrum discrets values of the voltage on the capacitor.

Let the entry conditions are zero, that is $X(0) = x(0) = 0$.

Having calculated by (3) necessary amount of discrets, we receive the decision as series, using retransformation (1).

III. Results

At calculation of the linear circuit (at $b = 0$) it is enough to use 20 discrets to receive exact enough solution (mean-square deviation is less than 10-11 %) on an interval of time 0…1 sec (Fig. 2, the solid line). However at increasing nonlinearly factor up to 0,2, it is already required 90 discrets to achieve sufficient accuracy of the restored signal (mean-square deviation is less than 0,001 %) (Fig. 2, the dotted line with a point). Attempt to restore the signal at nonlinearly factor $b = 2$ gives true result (Fig. 2, the thick dash line) only on an interval of time 0…0,4 sec, thus it is enough to use 40 discrets. From Fig. it is visible, that on an interval of time 0…0,4 sec the received solution visually precisely coincides with the true signal (Fig. 2, the thin dash line), received by Runge-Kutt method of the fourth order. To receive the signal on the greater interval of time it is necessary to use the greater amount of discrets at restoration [2]. However the increasing discrets amount does not give desirable result, even at use 6000 discrets.

The problem consists that at great values of nonlinearly factor the spectrum of discrets becomes quickly growing, and at signal restoration on the big time interval the role of discrete with higher number and their accuracy (on which accuracy of the restored signal depends) increases very much. At calculation of discrets by the recurrent formula with the limited digit grid there is an accumulation of a methodic error, which becomes the most essential with increasing discrete number. So that to receive the signal on the greater time interval it is necessary to use a lot of discrets, but using a lot of discrets does not give good results because of a methodical error at their calculation.

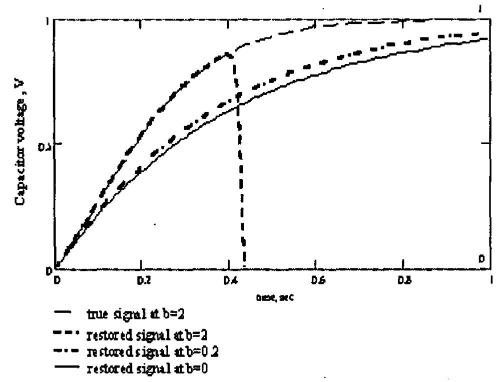

Fig. 2. The Transitive characteristic of a circuit at R=40 Ohm, C=10 mF, Emax=1 V.

IV. Conclusion

Calculations were carried out with the help of package MathCAD which uses accuracy of meaning digits 52 bit. Attempt to use accuracy of 66 bits at calculations by Pascal language does not give essential results. For cardinal increasing accuracy of results it is necessary to use the big accuracy arithmetic that it is possible to provide with software-based method. The solution of the given problem enables estimations of quality of parametrical synthesis of nonlinear electric circuits with the help of the differential transformations and a simple way to find transitive characteristics of nonlinear systems.

References

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